2.27.1 Normal Form Problems

A. For each of the following truth tables, provide a sentence in **Disjunctive Normal Form** which takes that truth table.

Truth Table 1

1
1
0
0
0
1
0
0

Truth Table 2

0
0
1
0
0
1
0
0

Truth Table 3

0
0
0
0
0
1
0
1

B. We noted that DNF imposes a strict **hierarchy of scope** on the connectives: a tilde always has smaller scope than a wedge or vel, and a wedge always has smaller scope than a vel.

Thanks to this scope hierarchy, we could leave out all parentheses in DNF without any sentence becoming ambiguous.

Assume that each of the following sentences is in DNF, and **add all missing parentheses** in order to yield a proper sentence of the Chapter Two language.

1. ~P ∨ Q ∧ P
2. P ∧ ~P ∨ P
3. ~P ∧ P ∨ Q ∧ ~Q
4. ~P ∨ P ∧ P

C. Build a truth table for each sentence from Problem (**B**) to decide whether that sentence is a **logical truth**, a **contradiction**, or **neither**.

D. State a general method which, for any given truth table, finds a matching sentence in **Conjunctive Normal Form**.

E. For each of the truth tables in Problem (**A**), state a matching sentence in **Conjunctive Normal Form**.

F. Decide, for each of the following sentences, if it's in **DNF**, **CNF**, or **neither**.

1. (~P ∧ Q) ∨ (P ∧ ~Q)
2. (P ∨ ~~Q) ∧ (~P ∨ Q)
3. (Q ∨ R ∨ S) ∧ (~Q ∨ ~R ∨ ~S)
4. ((Q ∨ R) ∧ S) ∨ ((~Q ∨ ~R) ∧ ~S)
5. (P ∨ (Q ∧ R)) ∧ (P ∨ (~Q ∨ ~R))
6. (~(Q ∨ R) ∧ S) ∨ ((~Q ∨ ~R) ∧ ~S)